PART III

SECTION – I

Straight Objective Type

This section contains 9 multiple choice questions numbered 45 to 53. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

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45. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}$, 2β be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is

(A) $\frac{2}{9}(p-q)(2q-p)$ (B) $\frac{2}{9}(q-p)(2p-q)$

(C) $\frac{2}{9}(q-2p)(2q-p)$ (D) $\frac{2}{9}(2p-q)(2q-p)$

46. Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$, and

$$\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$$

for each $x > 0$. Then $f(x)$ is

(A) $\frac{1}{3x} + \frac{2x^2}{3}$ (B) $\frac{-1}{3x} + \frac{4x^2}{3}$ (C) $\frac{-1}{x} + \frac{2}{x^2}$ (D) $\frac{1}{x}$

47. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{1}{5}$

48. The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining the points $(c-1, e^{c-1})$ and $(c+1, e^{c+1})$

(A) on the left of $x = c$ (B) on the right of $x = c$

(C) at no point (D) at all points

49. $$\lim_{x \to \frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)\, dt}{x^2 - \frac{\pi^2}{16}}$$ equals

(A) $\frac{8}{\pi} f(2)$ (B) $\frac{2}{\pi} f(2)$ (C) $\frac{2}{\pi} f\left(\frac{1}{2}\right)$ (D) $4 f(2)$

50. A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is

(A) $x^2 \operatorname{cosec}^2\theta - y^2 \sec^2\theta = 1$ (B) $x^2 \sec^2\theta - y^2 \operatorname{cosec}^2\theta = 1$

(C) $x^2 \sin^2\theta - y^2 \cos^2\theta = 1$ (D) $x^2 \cos^2\theta - y^2 \sin^2\theta = 1$

51. The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar, is

(A) zero (B) one

(C) two (D) three

52. A man walks a distance of 3 units from the origin towards the north-east $(N\ 45^\circ\ E)$ direction. From there, he walks a distance of 4 units towards the north-west $(N\ 45^\circ\ W)$ direction to reach a point P. Then the position of P in the Argand plane is

(A) $3e^{i\pi/4} + 4i$ (B) $(3 - 4i)e^{i\pi/4}$

(C) $(4 + 3i)e^{i\pi/4}$ (D) $(3 + 4i)e^{i\pi/4}$

53. The number of solutions of the pair of equations

$$2\sin^2\theta - \cos 2\theta = 0$$
$$2\cos^2\theta - 3\sin\theta = 0$$

in the interval $[0, 2\pi]$ is

(A) zero (B) one (C) two (D) four

SECTION – II

Assertion – Reason Type

This section contains 4 questions numbered 54 to 57. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

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54. Let $H_1, H_2, \cdots, H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, $i = 1, 2, \ldots, n$. Let E be any other event with $0 < P(E) < 1$.

STATEMENT-1 : $P(H_i \mid E) > P(E \mid H_i) \cdot P(H_i)$ for $i = 1, 2, \ldots, n$.

because

STATEMENT-2 : $\sum_{i=1}^{n} P(H_i) = 1$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

55. Tangents are drawn from the point $(17, 7)$ to the circle $x^2 + y^2 = 169$.

STATEMENT-1 : The tangents are mutually perpendicular.

because

STATEMENT-2 : The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

56. Let the vectors $\vec{PQ}$, $\vec{QR}$, $\vec{RS}$, $\vec{ST}$, $\vec{TU}$ and $\vec{UP}$ represent the sides of a regular hexagon.

STATEMENT-1 : $\vec{PQ} \times \left(\vec{RS} + \vec{ST}\right) \neq \vec{0}$.

because

STATEMENT-2 : $\vec{PQ} \times \vec{RS} = \vec{0}$ and $\vec{PQ} \times \vec{ST} \neq \vec{0}$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

57. Let $F(x)$ be an indefinite integral of $\sin^2 x$.

STATEMENT-1 : The function $F(x)$ satisfies $F(x + \pi) = F(x)$ for all real x.

because

STATEMENT-2 : $\sin^2(x + \pi) = \sin^2 x$ for all real x.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

SECTION – III

Linked Comprehension Type

This section contains 2 paragraphs **M58-60** and **M61-63**. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

M58-60 : Paragraph for Question Nos. 58 to 60

Let V_r denote the sum of the first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r-1)$. Let

$$T_r = V_{r+1} - V_r - 2 \text{ and } Q_r = T_{r+1} - T_r \text{ for } r = 1, 2, \ldots$$

58. The sum $V_1 + V_2 + \ldots + V_n$ is

(A) $\frac{1}{12}n(n+1)(3n^2 - n + 1)$ (B) $\frac{1}{12}n(n+1)(3n^2 + n + 2)$

(C) $\frac{1}{2}n(2n^2 - n + 1)$ (D) $\frac{1}{3}(2n^3 - 2n + 3)$

59. T_r is always

(A) an odd number (B) an even number

(C) a prime number (D) a composite number

60. Which one of the following is a correct statement?

(A) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 5

(B) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 6

(C) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 11

(D) $Q_1 = Q_2 = Q_3 = \ldots$

M61-63 : Paragraph for Question Nos. 61 to 63

Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and the fourth quadrants, respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S.

61. The ratio of the areas of the triangles PQS and PQR is

(A) $1 : \sqrt{2}$ (B) $1 : 2$ (C) $1 : 4$ (D) $1 : 8$

62. The radius of the circumcircle of the triangle PRS is

(A) 5 (B) $3\sqrt{3}$ (C) $3\sqrt{2}$ (D) $2\sqrt{3}$

63. The radius of the incircle of the triangle PQR is

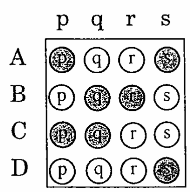
(A) 4 (B) 3 (C) $\frac{8}{3}$ (D) 2

SECTION – IV

Matrix-Match Type

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column I** have to be matched with statements (p, q, r, s) in **Column II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s, then the correctly bubbled 4×4 matrix should be as follows :



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64. Consider the following linear equations

$$ax + by + cz = 0$$
$$bx + cy + az = 0$$
$$cx + ay + bz = 0$$

Match the conditions/expressions in **Column I** with statements in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column I	**Column II**
(A) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(p) the equations represent planes meeting only at a single point.
(B) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(q) the equations represent the line $x = y = z$.
(C) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(r) the equations represent identical planes.
(D) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(s) the equations represent the whole of the three dimensional space.

65. In the following $[x]$ denotes the greatest integer less than or equal to x.

Match the functions in **Column I** with the properties in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column I	Column II
(A) $x\|x\|$	(p) continuous in $(-1, 1)$
(B) $\sqrt{\|x\|}$	(q) differentiable in $(-1, 1)$
(C) $x + [x]$	(r) strictly increasing in $(-1, 1)$
(D) $\|x-1\| + \|x+1\|$	(s) not differentiable at least at one point in $(-1, 1)$

66. Match the integrals in **Column I** with the values in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column I	Column II
(A) $\int_{-1}^{1} \frac{dx}{1+x^2}$	(p) $\frac{1}{2}\log\left(\frac{2}{3}\right)$
(B) $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$	(q) $2\log\left(\frac{2}{3}\right)$
(C) $\int_{2}^{3} \frac{dx}{1-x^2}$	(r) $\frac{\pi}{3}$
(D) $\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}}$	(s) $\frac{\pi}{2}$