PART III

SECTION – I

Straight Objective Type

This section contains 9 multiple choice questions numbered 45 to 53. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

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45. Let $O(0, 0)$, $P(3, 4)$, $Q(6, 0)$ be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are

(A) $\left(\frac{4}{3}, 3\right)$ (B) $\left(3, \frac{2}{3}\right)$

(C) $\left(3, \frac{4}{3}\right)$ (D) $\left(\frac{4}{3}, \frac{2}{3}\right)$

46. If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1 - z^2}$ lie on

(A) a line not passing through the origin

(B) $|z| = \sqrt{2}$

(C) the x-axis

(D) the y-axis

47. Let E^c denote the complement of an event E. Let E, F, G be pairwise independent events with $P(G) > 0$ and $P(E \cap F \cap G) = 0$. Then $P(E^c \cap F^c \mid G)$ equals

(A) $P(E^c) + P(F^c)$ (B) $P(E^c) - P(F^c)$

(C) $P(E^c) - P(F)$ (D) $P(E) - P(F^c)$

48. $\frac{d^2x}{dy^2}$ equals

(A) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (B) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

(C) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (D) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

49. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

(A) variable radii and a fixed centre at $(0, 1)$

(B) variable radii and a fixed centre at $(0, -1)$

(C) fixed radius 1 and variable centres along the x-axis

(D) fixed radius 1 and variable centres along the y-axis

50. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Which one of the following is correct?

(A) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} = \vec{0}$

(B) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq \vec{0}$

(C) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{a} \times \vec{c} \neq \vec{0}$

(D) $\vec{a} \times \vec{b},\ \vec{b} \times \vec{c},\ \vec{c} \times \vec{a}$ are mutually perpendicular

51. Let $ABCD$ be a quadrilateral with area 18, with side AB parallel to the side CD and $AB = 2\,CD$. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral $ABCD$ touching all the sides, then its radius is

(A) 3 (B) 2 (C) $\frac{3}{2}$ (D) 1

52. Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for $n \geq 2$ and $g(x) = \underbrace{(f \circ f \circ \cdots \circ f)}_{f \text{ occurs } n \text{ times}}(x)$. Then $\int x^{n-2} g(x)\, dx$ equals

(A) $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}} + K$

(B) $\frac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}} + K$

(C) $\frac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}} + K$

(D) $\frac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}} + K$

53. The letters of the word **COCHIN** are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word **COCHIN** is

(A) 360 (B) 192 (C) 96 (D) 48

SECTION – II

Assertion – Reason Type

This section contains 4 questions numbered 54 to 57. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

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54. Consider the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.

STATEMENT-1 : The parametric equations of the line of intersection of the given planes are $x = 3 + 14t,\ y = 1 + 2t,\ z = 15t$.

because

STATEMENT-2 : The vector $14\hat{i} + 2\hat{j} + 15\hat{k}$ is parallel to the line of intersection of given planes.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

55. STATEMENT-1 : The curve $y = \frac{-x^2}{2} + x + 1$ is symmetric with respect to the line $x = 1$.

because

STATEMENT-2 : A parabola is symmetric about its axis.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

56. Let $f(x) = 2 + \cos x$ for all real x.

STATEMENT-1 : For each real t, there exists a point c in $[t, t+\pi]$ such that $f'(c) = 0$.

because

STATEMENT-2 : $f(t) = f(t + 2\pi)$ for each real t.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

57. Lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

STATEMENT-1 : The ratio $PR : RQ$ equals $2\sqrt{2} : \sqrt{5}$.

because

STATEMENT-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

SECTION – III

Linked Comprehension Type

This section contains 2 paragraphs **M58-60** and **M61-63**. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

M58-60 : Paragraph for Question Nos. 58 to 60

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$, let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

58. Which one of the following statements is correct?
(A) $G_1 > G_2 > G_3 > \cdots$
(B) $G_1 < G_2 < G_3 < \cdots$
(C) $G_1 = G_2 = G_3 = \cdots$
(D) $G_1 < G_3 < G_5 < \cdots$ and $G_2 > G_4 > G_6 > \cdots$

59. Which one of the following statements is correct?
(A) $A_1 > A_2 > A_3 > \cdots$
(B) $A_1 < A_2 < A_3 < \cdots$
(C) $A_1 > A_3 > A_5 > \cdots$ and $A_2 < A_4 < A_6 < \cdots$
(D) $A_1 < A_3 < A_5 < \cdots$ and $A_2 > A_4 > A_6 > \cdots$

60. Which one of the following statements is correct?
(A) $H_1 > H_2 > H_3 > \cdots$
(B) $H_1 < H_2 < H_3 < \cdots$
(C) $H_1 > H_3 > H_5 > \cdots$ and $H_2 < H_4 < H_6 < \cdots$
(D) $H_1 < H_3 < H_5 < \cdots$ and $H_2 > H_4 > H_6 > \cdots$

M61-63 : Paragraph for Question Nos. 61 to 63

If a continuous function f defined on the real line **R**, assumes positive and negative values in **R** then the equation $f(x) = 0$ has a root in **R**. For example, if it is known that a continuous function f on **R** is positive at some point and its minimum value is negative then the equation $f(x) = 0$ has a root in **R**.

Consider $f(x) = ke^x - x$ for all real x where k is a real constant.

61. The line $y = x$ meets $y = ke^x$ for $k \leq 0$ at
(A) no point (B) one point
(C) two points (D) more than two points

62. The positive value of k for which $ke^x - x = 0$ has only one root is
(A) $\frac{1}{e}$ (B) 1 (C) e (D) $\log_e 2$

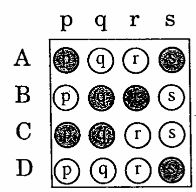
63. For $k > 0$, the set of all values of k for which $ke^x - x = 0$ has two distinct roots is
(A) $\left(0, \frac{1}{e}\right)$ (B) $\left(\frac{1}{e}, 1\right)$ (C) $\left(\frac{1}{e}, \infty\right)$ (D) $(0, 1)$

SECTION – IV

Matrix-Match Type

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column I** have to be matched with statements (p, q, r, s) in **Column II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s, then the correctly bubbled 4×4 matrix should be as follows :



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64. Let $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 5x + 6}$.

Match the expressions/statements in **Column I** with expressions/statements in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column I	**Column II**
(A) If $-1 < x < 1$, then $f(x)$ satisfies	(p) $0 < f(x) < 1$
(B) If $1 < x < 2$, then $f(x)$ satisfies	(q) $f(x) < 0$
(C) If $3 < x < 5$, then $f(x)$ satisfies	(r) $f(x) > 0$
(D) If $x > 5$, then $f(x)$ satisfies	(s) $f(x) < 1$

65. Let (x, y) be such that

$$\sin^{-1}(ax) + \cos^{-1}(y) + \cos^{-1}(bxy) = \frac{\pi}{2}.$$

Match the statements in **Column I** with statements in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	**Column II**
(A) If $a = 1$ and $b = 0$, then (x, y)	(p) lies on the circle $x^2 + y^2 = 1$
(B) If $a = 1$ and $b = 1$, then (x, y)	(q) lies on $(x^2 - 1)(y^2 - 1) = 0$
(C) If $a = 1$ and $b = 2$, then (x, y)	(r) lies on $y = x$
(D) If $a = 2$ and $b = 2$, then (x, y)	(s) lies on $(4x^2 - 1)(y^2 - 1) = 0$

66. Match the statements in **Column I** with the properties in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	**Column II**
(A) Two intersecting circles	(p) have a common tangent
(B) Two mutually external circles	(q) have a common normal
(C) Two circles, one strictly inside the other	(r) do not have a common tangent
(D) Two branches of a hyperbola	(s) do not have a common normal